4.4 Irrational Numbers

Integers
- Positive
- Negative
- Including 0

Natural numbers (Counting numbers)

Whole numbers (0 and Natural numbers)

Rational Numbers
- Any number that can be written as a fraction or terminating or repeating decimal.

Irrational Numbers
- Cannot be written as a terminating or repeating decimal. Ex: \( \sqrt{3}, \pi, \frac{\pi}{3} \)

Radical: \( \sqrt[n]{x} \) can be written as a power: \( x^{\frac{1}{n}} \)

Note: \( (9^{\frac{1}{2}})(9^{\frac{1}{2}}) = 9^1 \) • Add exponents.

Also \( (\sqrt{9})(\sqrt{9}) = (3)(3) = 9^1 \)

So \( 9^{\frac{3}{2}} = \sqrt{9^3} \)

• When it is square root, the index 2 is usually assumed.

In general: \( x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m = \sqrt[n]{x^m} \)

Example 1

a) \( 64^{\frac{1}{2}} = \sqrt{64} = 8 \)

b) \( 16^{\frac{3}{4}} = \sqrt[4]{16^3} = 2^3 = 8 \)

* It is easier if you can calculate the root first. \( \sqrt[4]{16} = 2 \) Since \( 2^4 = 16 \).
c) \((8x^2)^{\frac{1}{3}} = \sqrt[3]{8x^2} = \sqrt[3]{8} \cdot \sqrt[3]{x^2} = 2 \sqrt[3]{x^2}\)

Try these: Rewrite each power as a radical:

d) \(10^{\frac{1}{4}}\) e) \(1024^{\frac{1}{3}}\) f) \((x^4)^{\frac{3}{8}}\)

Example 2: Express each radical as a power.

a) \(\sqrt[4]{4^3} = 4^{\frac{3}{4}}\) b) \(\sqrt[3]{3^4} = 3^{\frac{4}{3}}\) c) \(\sqrt[5]{5^3} = 5^{\frac{3}{5}}\)

Try these d) \(\sqrt[5]{125}\) e) \(\sqrt[5]{125}\) f) \(\sqrt[5]{27^2}\)