Chapter 7 Test

Multiple Choice

For #1 to #6, choose the best answer.

1. What is the \( y \) -intercept for the graph of \( y = b^{x-2}, \; b > 1 \)?
   \[ \begin{align*}
   A & \quad \frac{-1}{b^3} \\
   B & \quad -b^2 \\
   C & \quad \frac{1}{b^3} \\
   D & \quad 2
   \end{align*} \]

2. In the equation \( y = b^x, \; b > 1 \), \( x \) is replaced by \( x - 3 \) and \( y \) is replaced by \( y - 4 \). Which of the following statements describes the transformation?
   \[ \begin{align*}
   A & \quad \text{The point } (x, y) \text{ on the graph of } y = b^x \text{ has been transformed to the point } (x + 3, y + 4).
   \\
   B & \quad \text{The point } (x, y) \text{ on the graph of } y = b^x \text{ has been transformed to the point } (x - 3, y - 4).
   \\
   C & \quad \text{The graph of } y = b^x \text{ has been translated 4 units to the right and 3 units up.}
   \\
   D & \quad \text{The graph of } y = b^x \text{ has been translated 3 units to the left and 4 units down.}
   \end{align*} \]

3. The graph of \( f(x) = a^x, \; a > 1 \), is transformed into \( g(x) = 4a^{x+3} - 2 \). Which characteristic remains the same?
   \[ \begin{align*}
   A & \quad \text{domain} \\
   B & \quad \text{range} \\
   C & \quad \text{x-intercept} \\
   D & \quad \text{y-intercept}
   \end{align*} \]

4. The graph of the function \( f(x) = 3a^x + 2, \; a > 0 \), has the same horizontal asymptote as which of the following?
   \[ \begin{align*}
   A & \quad y = -f(x) - 4 \\
   B & \quad y = -f(x) - 2 \\
   C & \quad y = -f(x) + 2 \\
   D & \quad y = -f(x) + 4
   \end{align*} \]

5. Mary was asked to solve for \( x \) and \( y \) in the exponential equations \( 5^x + 3^y = 1 \) and \( 25^x + 5^y = \frac{1}{5} \). Which of the following linear equations would lead to a correct solution?
   \[ \begin{align*}
   A & \quad x + 3y = 1, \; x + y = -1 \\
   B & \quad x + 3y = 0, \; 2(x + y) = -1 \\
   C & \quad x + 3y = 1, \; 2x + y = -1 \\
   D & \quad x + 3y = 0, \; x + y = -2
   \end{align*} \]

6. Which function(s) would you graph to solve the equation \( 16 \cdot \frac{1}{4} = \left( \frac{1}{2} \right)^{4x + 3} \) graphically?
   \[ \begin{align*}
   A & \quad y_1 = 16^{x - 0.5}, \; y_2 = 0.5^{4x + 3} \\
   B & \quad y_1 = 16^{x - 0.5} - \left( \frac{1}{2} \right)^{4x + 3} \\
   C & \quad y_1 = \left( \frac{1}{2} \right)^{4x + 3} + 16^{2x} \\
   D & \quad y_1 = 4x, \; y_2 = \left( \frac{1}{2} \right)^{4x + 3}
   \end{align*} \]

Short Answer

7. Given the function \( f(x) = 2^x \), match the graph with the correct equation.
   \[ \begin{align*}
   a) & \quad y = -f(x) \\
   b) & \quad y = f(-x) \\
   c) & \quad y = f^{-1}(x) \\
   d) & \quad y = -f(-x)
   \end{align*} \]
8. The function \( f(x) = -5(2^x) \) is transformed by a translation 2 units right and 5 units down. The transformed function passes through the point \((x, -10)\). Determine the value of \( x \).

9. What vertical translation would be applied to \( y = 4(3^x) \) so that the translation image passes through \((2, 37)\)?

10. Solve for \( x \).
   \[ \frac{x}{3^2} = 81 \sqrt{3} \]
   \[ \left( \frac{9}{16} \right)^{x+2} = \left( \frac{64}{27} \right)^x \]

Extended Response

11. You are given the functions \( y = 2^{-x} \) and \( y = 2(2^{-x}) - 3 \).
   a) Sketch the graphs of the functions on the same grid.
   b) Describe the transformation from \( y = 2^{-x} \) to \( y = 2(2^{-x}) - 3 \).
   c) State the range and the equation of the horizontal asymptote for each function.
   d) Determine the value of \( y \) when \( x = 400 \) for each function. Explain how these results relate to your answers to part c).

12. Consider the graph of the functions \( f \) and \( g \).
   a) Determine the equation of the transformed function \( g(x) \).
   b) Describe the transformation of \( f(x) \) to \( g(x) \).
   c) Use the graphs to solve the equation \( f(x) = g(x) \), to the nearest hundredth.

13. A single cell of the bacterium \( E. coli \) would, under ideal circumstances, divide every 20 minutes.
   a) If a culture begins with 1 bacterium, write the equation for the number of bacteria after \( n \) minutes.
   b) Determine, to the nearest minute, the time it takes for the culture to grow to 1024 bacteria.
   c) If each bacterium has a mass of roughly \( 10^{-12} \) g, what is the mass of the bacteria after 1 day, to the nearest kg?

14. A town had a population of 2200 people in 1990. Each year the population has decreased by 10%.
   a) Write an equation to represent the population of the town.
   b) What will the population be in the year 2020?
   c) When will the population be less than 50 people?