

Chapter 8 Test

Multiple Choice

For #1 to 6, select the best answer.

1. The graph of \( f(x) = \log_b x \), \( b > 1 \), is translated such that the equation of the new graph is expressed as \( y - 2 = f(x - 1) \). The domain of the new function is
   A \( \{ x | x > 0, x \in \mathbb{R} \} \)
   B \( \{ x | x > 1, x \in \mathbb{R} \} \)
   C \( \{ x | x > 2, x \in \mathbb{R} \} \)
   D \( \{ x | x > 3, x \in \mathbb{R} \} \)

2. The \( x \)-intercept of the function \( f(x) = \log x + 3 \) is
   A \( 5^{-3} \)
   B \( 0 \)
   C \( 1 \)
   D \( 5^3 \)

3. The equation \( y = \frac{1}{3} \log x \) can also be written as
   A \( y = 2^x \)
   B \( x = 2^y \)
   C \( 2^{3x} = y \)
   D \( 2^{3y} = x \)

4. The range of the inverse function, \( f^{-1} \), of \( f(x) = \log_b x \), is
   A \( \{ y | y > 0, y \in \mathbb{R} \} \)
   B \( \{ y | y < 0, y \in \mathbb{R} \} \)
   C \( \{ y | y > 0, y \in \mathbb{R} \} \)
   D \( \{ y | y \in \mathbb{R} \} \)

5. A graph of the function \( y = \log_b x \) is transformed. The image of the point (3, 1) is (6, 3). The equation of the transformed function is
   A \( y = 3 \log_b (x - 3) \)
   B \( y = 3 \log_b (x + 3) \)
   C \( y - 3 = \log_b (x - 3) \)
   D \( y + 3 = \log_b (x + 3) \)

6. If \( \log_3 x = \frac{y}{3} \), then \( \log_b x \) equals
   A \( \frac{2y}{2} \)
   B \( \frac{2y}{3} \)
   C \( 3y \)
   D \( 4^y \)

Short Answer

7. If \( \log_3 5 = x \), express \( 2 \log_3 45 - \frac{1}{2} \log_3 225 \) in terms of \( x \).

8. Determine the value of \( x \) algebraically.
   a) \( \log_2 x = -3 \)
   b) \( \log_8 64 = \frac{2}{3} \)
   c) \( 5 \cdot 2^x = x \)
   d) \( \log_3 (x + 1)^2 = 2 \)
   e) \( \log_3 (\log_2 256) = 3 \)

9. Solve for \( x \).
   a) \( \log (2x - 3) + \log (x - 2) = \log (2x - 1) \)
   b) \( \log (x - 7) - \log (x - 3) = \log (2x + 1) \)
   c) \( 2 \log_5 (x - 4) - \log_5 x = 1 \)

10. The point (6, -4) lies on the graph of \( y = \log_b x \). Determine the value of \( b \) to the nearest tenth.

Extended Response

11. Solve the equation \( 5^x = 104 \), graphically and algebraically. Round your answer to the nearest hundredth.

12. Given \( f(x) = \log_b x \) and \( g(x) = \log_b 9x \).
   a) Describe the transformation of \( f(x) \) required to obtain \( g(x) \) as a stretch.
   b) Describe the transformation of \( f(x) \) required to obtain \( g(x) \) as a translation.
   c) Determine the \( x \)-intercept of \( f(x) \). How can \( x \)-intercept of \( g(x) \) be determined using your answer to parts a) or b)?

13. Explain how the graph of \( y = \frac{\log_3 (3x - 1) + 1}{2} \) can be generated by transforming the graph of \( y = \log_b x \).

14. Identify the following characteristics of the graph of the function \( y = 2 \log_b (x + 1) + 3 \).
   a) the equation of the asymptote
   b) the domain and range
   c) the \( x \)-intercept and the \( y \)-intercept

15. An investment of \$2000 pays interest at a rate of 3.5% per year. Determine the number of months required for the investment to grow to at least \$3000 if interest is compounded monthly.

16. Radioactive iodine-131 has a half-life of 8.1 days. How long does it take for the level of radiation to reduce to 1% of the original level? Express your answer to the nearest tenth.